Fraction Problems

Sources:

Using Math to Teach Math, Mark Hoover Thames, MSRI, 2007.

Nat'l Council of Teachers of Mathematics

Karl Schaffer
Math 46
De Anza College

For each of these fractions, determine whether it is closest to 0, 1/2, or 1. Use one of the techniques we practiced in the last class.

$$\frac{3}{8}$$

$$\frac{5}{7}$$

$$\frac{3}{13}$$

Which is larger?
Use the methods from last class to decide:

$$\frac{5}{9} \; or \; \frac{4}{7}?$$

$$\frac{3}{8} \; or \; \frac{6}{17}?$$

$$\frac{5}{9} + \frac{3}{7} \;\; or \;\; 1?$$

Put in order, smallest to largest, doing no paper and pencil calculations.

$$\frac{3}{4}, \frac{1}{4}, \frac{7}{8}, \frac{1}{12}, \frac{4}{5}, \frac{1}{16}, \frac{2}{3}, \frac{5}{7}$$

Sylvia and Delia cut a rectangular lawn. The part Delia mowed is 3/2 as large as the part Sylvia mowed.

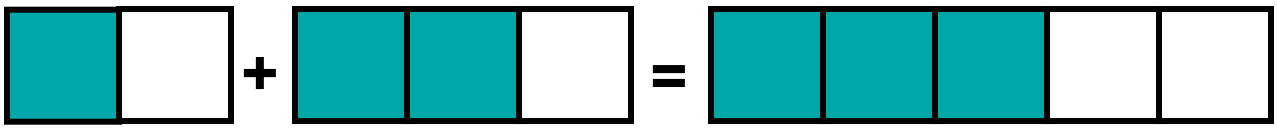
(a) Cut the rectangle to show how the lawn was cut.
(b) The part Sylvia mowed is _____ as large as the part Delia mowed.
(c) The were given $25 for mowing the lawn; how should the money be divided? Explain

Fraction Addition

Jessica, a fifth grader, says that:

$$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$$

As proof she offers the following picture.



What is the problem with Jessica's reasoning?

Using Jessica's way of representing fractions, what picture could you show her that correctly Represents the additon in question?

- Beckman, S. (2003). *Mathematics for Elementary Teachers.*

Baseball Addition

Sat. May 19, 2007 Doubleheader between Atlanta Braves and Boston Red Sox

Game 1:
Manny Ramirez: 2 for 4, average of 2/4
Mike Lowell: 4 for 5, average of 4/5
David Ortiz: 3 for 3, average of 3/3

Game 2:
Manny Ramirez: 0 for 2, average of 0/2
Mike Lowell: 1 for 3, average of 1/3
David Ortiz: 0 for 3, average of 0/3

Why does the "baseball addition" method give the correct overall two-game average for each player?
Explain what is going on!

$$\frac{2}{4}+\frac{0}{2}=\frac{2}{6}? \qquad \frac{4}{5}+\frac{1}{3}=\frac{5}{8}? \qquad \frac{3}{3}+\frac{0}{3}=\frac{3}{6}?$$

Farey Sequences

British geologist John Farey, 1816

Mathematician C. Haros, 1802

This chart produces all the fractions between 0 and 1, already in lowest terms, using “baseball addition.” Can you explain how the chart works? (It is similar to Pascal’s triangle!)

$$\frac{0}{1} \qquad \frac{1}{1}$$

$$\frac{0}{1} \qquad \frac{1}{2} \qquad \frac{1}{1}$$

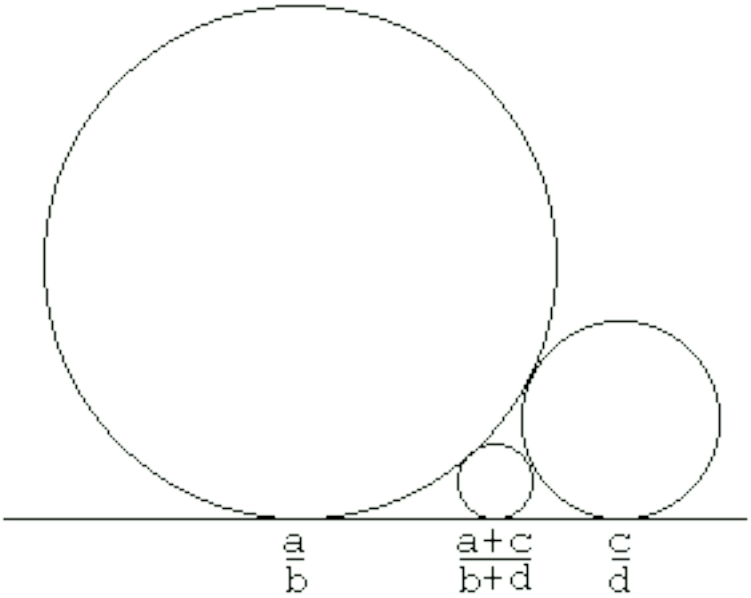
$$\frac{0}{1} \qquad \frac{1}{3} \qquad \frac{1}{2} \qquad \frac{2}{3} \qquad \frac{1}{1}$$

$$\frac{0}{1} \qquad \frac{1}{4} \quad \frac{1}{3} \qquad \frac{1}{2} \qquad \frac{2}{3} \quad \frac{3}{4} \qquad \frac{1}{1}$$

$$\frac{0}{1} \quad \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{2}{5} \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{1}{1}$$

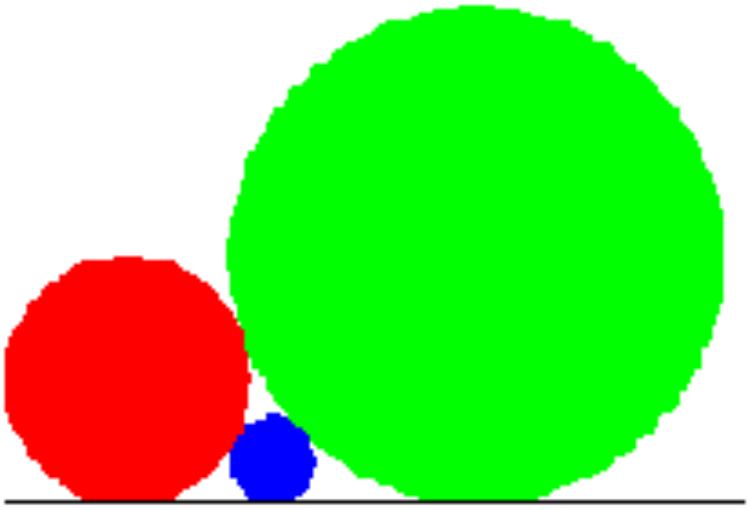
Mediants and Ford Circles

$$\text{"}Mediant\text{"}\ \ of\ \ \frac{a}{b}\ and\ \frac{c}{d}\ \ is\ \ \frac{a+c}{b+d}$$



“Baseball addition” (or the “mediant”) produces the locations on the number lines of the tangent points of the so-called Ford circles
- Lester Ford, 1938

Japanese Sangaku problems



Japan, circa 1800, hung as tablets in Buddhist temples:Found the radii of mutually tangent circles, prior to the discovery of “mediant” process in the West.

Draw a diagram which explains why

$$\frac{2}{5} \circ \frac{3}{7} = \frac{2 \circ 3}{5 \circ 7}$$

Fraction Division

David calculated

$$\frac{8}{15} \; divided \;\; by \;\; \frac{2}{3}$$

**as follows: “8 divided by 2 is 4,
and 15 divided by 3 is 5,
so the answer is 4/5.”**

Is David’s answer correct?

Is David’s method correct?

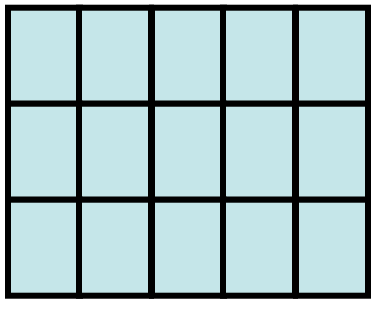
Is this a generally applicable method?

Using Math to Teach Math, **Mark Hoover Thames, MSRI, 2007.**

Comparing Fractions

For a lesson comparing fractions, Ms. Banks wants to choose a model that will make it easy for her students to compare a wide range of fractions. What would be the advantages and disadvantages of using drawings of circles? Drawings of rectangles? Money?

How might the 3 by 5 rectangle below be used to compare 2/5 and 1/3?



Using Math to Teach Math, **Mark Hoover Thames, MSRI, 2007.**

Fraction Multiplication

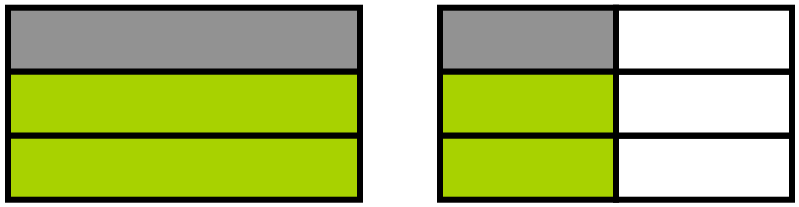
At a workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also had them consider examples that do not represent multiplication of fractions appropriately. Which models below can be used to show that

$$1\frac{1}{2} \times \frac{2}{3} = 1?$$

For each you select explain how it is interpreted.

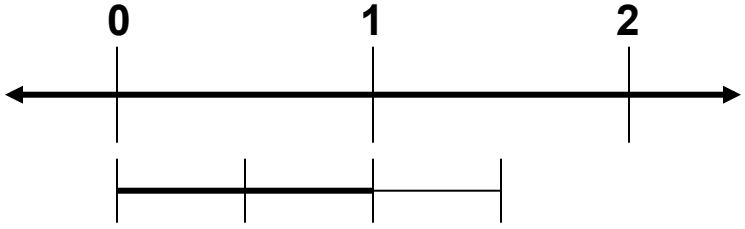
A)

B)



C)

D)



Using Math to Teach Math, **Mark Hoover Thames, MSRI, 2007.**

“Story” Problems

Write a story problem that is modeled by the mathematical expression:

$$1\frac{1}{4} \div \frac{1}{2}$$

Make a geometric representation of the expression and show how it represents each part of the expression.

Using Math to Teach Math, **Mark Hoover Thames, MSRI, 2007.**